Topic Study Group 58

Empirical Methods and Methodologies in Mathematics Education

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ABSTRACT In TSG-58 research methods, methodologies, and paradigms related to traditional issues of mathematics education such as instruction, learning, teaching and classroom processes and interactions were discussed. In total twelve papers and two posters were presented and discussed over three sessions. Overall, about 50 scholars participated in this TSG-58.

Keywords: Methods; Methodologies; Paradigms.

Research in mathematics education employs a range of Methods, Methodologies, and Paradigms (M/M/Ps) in the service of key goals. TSG-58 promoted a discussion about diverse strands of M/M/Ps investigating these goals.

1. Methods, Methodologies, and Paradigms in the Service of Key Goals

In the call for TSG-58 six diverse goals central to ongoing research in mathematics education were promoted, but three of them — Mathematics Education and Social Justice, the Role of Culture and Language in Shaping the Teaching and Learning of Mathematics — were not in focus of the submitted papers. Instead, traditional issues as instruction, learning, teaching in general and classroom processes and interactions were explicitly discussed in the submitted methodical and methodological papers of our TSG-58.

1.1. Diverse goals central to ongoing research in mathematics education

TSG-58 was finally organised, in three sessions, around four diverse goals central to ongoing research in mathematics education:

1) Improvement of Mathematics Instruction (e.g., instructional materials, strategies, organization, assessment);
2) Learning of Mathematics;

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3) Teaching of Mathematics (e.g., teacher beliefs, knowledge, decision-making and professional development); and
4) Classroom Processes and Interactions

Each *goal* was addressed using research designs that integrate one or more different Methods, Methodologies, and Paradigms (M/M/Ps). For each *goal*, the contributors of TSG-58 were asked to address the following questions and to discuss which M/M/P combinations help us understand the phenomena at stake in robust and reliable ways.

1) "Suppose you have a hypothesis about this goal. How do you set about evaluating it?" Alternatively,
2) "Suppose you are trying to explain some aspect of individual or group behavior relevant to that goal. How would you characterize and then theorize that behavior?"
3) Or, "How might cultural, historical and political perspectives shape one's understandings of the contingencies related to realizing this particular goal?"

The *goals* of our research clearly in focus, the actual topic of TSG-58 were the different methods, methodologies, and paradigms (M/M/Ps) employed.

1.2. Empirical research methods and methodologies

This TSG was specifically focused on the empirical research methods and methodologies employed to address the four broad goals of research in mathematics education identified above. For our work to be coherent and allow for comparability, each paper identified the specific goal(s) being explored, identified the theoretical frame on which the research design was predicated, and addressed the question of how effectively the research design (M/M/P bundle) addressed the designated goal(s). Participants were asked to

1) Specify the methodology and methods that constitute the research design and identify the particular goal/s that are the focus of the reported research study;
2) Specify the theoretical frame or rationale by which the selection of methodology and methods can be justified, discussing advantages and limitations of methodological choices respectively the identified research goal(s);
3) Further address the appropriateness of the chosen methods in terms of the robustness of the findings generated, their generality or specific domain of relevance, and their capacity to describe, explain or predict phenomena of importance to the field of mathematics education.

The following brief summaries of the papers in our TSG-58 only indicate the theoretical frames, research designs and designated goals of each contribution. Longer papers about the reported research are published elsewhere and indicated to in the references.

2. Program Overview

Tab. 1. List of papers and posters presented

Paper and author(s)
Session 1 (July 13th at 19:30 – 21:00 Beijing Time)
[1] First voyage of the integrated paradigm: The case of an international study on effective mathematics teaching. ***Zhenzhen Miao*** (China), *and David Reynolds and Christian Bokhove* (UK).
[2] The teaching of mathematical thinking: The conceptualization of a special class teacher in China. ***Na Li*** (China) *and Ida Ah Chee Mok* (Hong Kong SAR, China).
[3] Teaching design of combination from HPM perspective. ***Weiyuan Fan*** (China).
Session 2 (July 16th, 21:30-23:00 Beijing Time)
[4] Understanding the relations between instructional quality and task quality in mathematics classrooms. ***Ann-Kristin Adleff***, *Natalie Ross, Gabriele Kaiser, Johannes König* (Germany), *and Sagrid Blömeke* (Norway).
[5] What is six-questions cognitive model? *Ying Zhou, Xiaofeng Lan, and* ***Tommy Tanu Wijaya*** (China).
[6] Units coordination as a theoretical construct to understand students mathematical activities. ***Soo Jin Lee*** *and Jaehong Shin* (South Korea).
[7] The influence of ICT on the students' science literacy at the national and student level based on ITU IDI Index and PISA2015. ***Zhenrong Xiong***, *Ying Zhang, Bo Li, and Na Li* (China).
[8] The effectiveness of teaching mathematics in circle equation by using 5E instructional model in inquiry-based learning. ***Try Kimhor*** (Cambodia). (Poster)
[9] The trend of mathematics teaching method has changed from fragments to systematics. *Yi Lin*, ***Tommy Tanu Wijaya***, *and Ying Zhou* (China) (Poster).
Session 3 (July 17th, 14:30-16:30 Beijing Time)
[10] Eye movements and collaborative problem solving: what do long fixations tell about student cognition? ***Markku S. Hannula***, *Enrique Garcia Moreno-Esteva, and Miika Toivanen* (Finland).
[11] Examining the phenomenon of interlocutors talking past each other in collaborative proof constructions. ***Ann Sophie Stuhlmann*** (Germany).
[12] Using MRGQAP to analyse the development of mathematics pre-service trainees' communication networks. ***Christian Bokhove*** (UK), *Jasperina Brouwer* (Netherland), *and Chris Downey* (UK).
[13] Case study of personalized teaching based on the Q-learning algorithm in the era of big data. ***Lei Wang***, *Yong Zhang, Na Li, Bo Li* (China).
[14] Learning research in a laboratory classroom: Advancing methodology and technology. ***Man Ching Esther Chan***, *and David Clarke* (Australia).

2.1. Session 1: Teaching

In our first session, Christine Knipping (Germany) and Soo Jin Lee (Korea) opened the TSG-58 with an introduction. They highlighted the diverse goals of research, listed above, and promoted the diverse strands of the M/M/P bundle as a way to address these designated goal(s), in terms of diverse methods, methodologies and paradigms. As the leaders of TSG-58 they proposed a programme, which is also used in this paper to structure the contributions of TSG-58. The first session started with papers on M/M/P issues around *Teaching*.

Miao et al.[1] reflected on a mixed methods approach as a way of researching teaching in an international setting (Miao and Reynolds, 2018). The title of the submission indicates a quantitative and qualitative methods approach that is designed

as a mixed methods comparative approach to gain deeper insights into issues around teaching and learning mathematics.

Li and Mok[2] presented a paper on research issues around teaching of mathematical thinking. Both researchers made overt that understanding (methodologically) teachers' perspectives on mathematical thinking is important to not only describe and analyze their views on the issue, but also the impact on teaching.

In her presentation[3] Fan (China) talked about teaching integrating historical materials into the classroom. Research and methodological issues are related to specific traditions and cultural issues in this paper.

2.2. Session 2: Instruction and learning

In the second session of TSG-58 papers on M/M/P issues were focusing on issues at the intersection of *Instruction and Learning*. How to research learning in the context of specific materials and designed activities were of particular interest in this session.

The presentation[4] by Adleff et al. explored how quantitative research methods can capture and assess the "instructional quality" in classrooms. They also discussed how the "task quality" can be measured and put in relation to the performed "instructional quality" in class (see also Kaiser et al., 2017).

Zhou et al.[5] proposed in their presentation a 6-questions model, which has been developed in China about 10 years ago, combined with technology-based learning media and reflect in how far this cognitive model can foster students' deep learning (see also Lin et al., 2020).

Lee and Shin[6] discussed how far the theoretical construct "Unit Coordination" can be used to understand mathematical activities of students, portrayed as "cognizing subjects" (see also Lee and Shin, 2021).

Xiong et al.[7] also investigated the role of information and communication technology (ICT) in the context of instruction/learning. They wondered what impact ICT developments have on students' scientific literacy.

Two poster presentations, Kimhor[8] and Lin et al.[9], were part of TSG-58. These posters deal with methodical issues around instruction, learning and teaching. The first poster looks into the effectiveness of teaching in the context of inquiry-based learning, based on an instructional model, while the second one proposes a "systematic plan of teaching mathematics", based on the so called "Dick-Carey" model which aims to offer teachers a systematic approach to mathematics teaching. They report in how far mathematics teaching methods have changed in China.

2.3. Session 3: Learning, teaching and the social dimension

In the last session of our TSG-58 even more diverse methodical and methodological facets were brought up and discussed. Students' cognition "as it happens" was looked at, as well as the phenomena of interaction and how these can divert. Also, social networks of peer pre-service mathematics trainees and the methods and theoretical approaches how to research these were presented and discussed. Two further contributions mirrored how diverse and multi-faceted these discussions were. A

"personalized teaching intervention" based on a "Q-learning algorithm", conceptualized as a "dynamic optimization problem" was presented and discussed as well as "Learning research in a laboratory classroom", where methodology and technology was designed so that the investigation of social aspects of classroom practice, particularly student-student and student-teacher interactions could be researched.

Hannula et al.[10] discussed how far paper and GeoGebra contexts effect fixation durations in collaborative student activities in geometry. From their observations and categorizations of four different types of long fixations they conclude possible cognitive student activities (see also Hannula and Toivanen, 2019; Hannula, Toivanen and Garcia Moreno-Esteva, 2019).

Stuhlmann[11] investigated students in collaborative proving activities in an undergraduate linear algebra class. Her interactionist methodology and methods allow her to study the diversity of meaning making of students in the same undergraduate class and why it is challenging for the students to come to a consensus during their proving process.

Bokhove et al.[12] the potential of a specific data analysis method (MRQAP) for analyzing longitudinal network data, in order to study the development of peer networks of pre-service mathematics trainees over time (see also Bokhove and Downey, 2018).

Wang et al.[13] presented a personalized teaching intervention, which is aimed to maximize academic performance of students (see also Wang et al., 2013). According to the researchers the latest advances in information technology allow this approach.

Last, but not least, Ching and Chan[14], based on collaboration with David Clarke, presented a multi-theoretic and multimodal research design, implemented in the laboratory classroom at the University of Melbourne (see Chan and Clarke, 2016, 2017). She discussed the complexity of this research design, which focused on student-student and student-teacher interactions.

1. Future Directions and Suggestions

The methodology and methods that constitute the research designs presented in the TSG-58 were not only diverse and multifaceted, but also indicated distinct and specific goals.

For example, *Teaching* was in some contributions not only researched in mixed methods ways, but also with an international comparative focus. Whereas, other research focused consciously on one cultural context only to deeper investigate the rationale of this specific context and historical tradition. Also, related to *Instruction and Learning* different goals were of interest. Established instruction approaches and models were valued in a methodically pragmatic way, i.e. within a new technology-based environment, and when looking at the impact of ICT environments on students' learning. On the other hand, more theoretical stances were taken to better understand mathematical activities of students.

Also *Learning, Teaching and the Social dimension* were studied not only in diverse methodical and methodological settings, but also with different goals. Understanding individual student cognition and academic performance was of interest for some researchers, other scholars focused clearly on the complexity of collaborative settings for students learning, as well as on student-student and student-teacher interactions.

The appropriateness of the chosen methods and theoretical frame or rationale by which the selection of methodology and methods were justified, was overtly discussed in the three TSG-58 sessions. Questions and comments highlighted advantages and limitations of methodological choices respectively the identified research goal(s). This helped to evaluate the robustness of the findings generated, their generality or specific domain of relevance, and their capacity to describe, explain or predict phenomena of importance to the field of mathematics education.

TSG-58 promoted a discussion about diverse strands of M/M/Ps investigating specific goals. Traditional issues as Instruction, Learning, Teaching in general and Classroom Processes and Interactions were explicitly discussed in the submitted methodical and methodological papers of our TSG-58. Extended discussions of Methods, Methodologies, and Paradigms (M/M/Ps) in research in mathematics education, in the service of key goals, will further substantiate the state of our art in the future. But these discussions will have to also include goals of ongoing research in the area of Mathematics Education and Social Justice, the Role of Culture and Language, shaping the Teaching and Learning of Mathematics, which were not in focus of the submitted papers.

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